International Theoretical Physics Olympiad - 2016

January 30, 2016

You have 24 hours to solve the problems, you can use any textbooks and online materials published before the beginning of the contest. You can also use Computer Algebra Systems like Mathematica or Maple. However, in case you want to present solution that relies on these computations please provide a complete description and all supplementary materials including the source code.

You can upload your solutions using your team page. Although we can accept almost all possible formats, the .pdf is the preferred one. Good luck!

1 Springs

Consider five mass points on the plane located in four edges and the center of a square with side length L. Each mass point has mass m and is attached with springs to another mass points and four fixed points as shown on Fig. 1. All strings are non-stretched at the initial position. Each of the eight diagonal springs has stiffness k, whereas all four horizontal and vertical strings have stiffness $2k$.

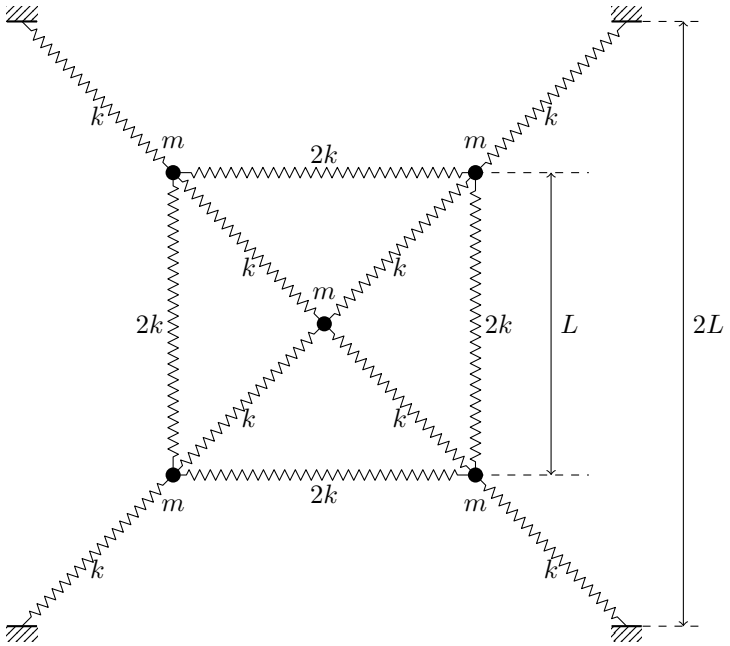

Figure 1: Springs arrangement

Considering the system as 2-dimensional:

1. Find all resonant frequencies for the classical system described above.
2. Consider quantum system with the same potential. Approximate five lowest energy levels for this system under assumption $mkL^4 \gg \hbar^2$.

2 Monopole

Consider a particle with mass m and charge e moving in a field of a magnetic monopole with magnetic charge g. Obtain the general classical non-relativistic trajectory of the particle and solve the scattering problem when the particle approaches the monopole with some initial velocity v_0, find the angle by which it would scatter. The impact parameter is fixed by the initial total angular momentum J of the system.

3 Casimir Force

Calculate the Casimir force acting between two parallel perfectly conducting planes with massive bosonic field ($E = \sqrt{p^2 + m^2}$) inside and outside the planes.

4 Radiation Reaction Force

Consider the theory of massive electrodynamics with the action

$$S = \int \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + A_\mu J^\mu \right) d^4x \,.$$

Suppose there is an external force acting on a point-like charge resulting in its motion along some general trajectory. Calculate the back-reaction force caused by the field of the charge, work out the example of circular motion. Reproduce the usual electrodynamics answer for the radiation reaction force by the massless photon limit. Repeat the same consideration for the force in terms of quantum field theory and compare your results.

5 Astronaut on a chain

Consider a space station of mass M on the orbit of radius R around a planet of mass $M_0 \gg M$. In open space near the station there is an astronaut of mass $m \ll M$. The astronaut is attached to the station via the chain that consists of $N \gg 1$ links each of length a. Links can be considered as rigid rods that are connected at their ends and can freely rotate one around another. The chain is reasonably short with the total length $Na \ll R$. Assuming that microwave background radiation keeps the system at temperature T, find the distance from the astronaut to the station.

6 Landau Levels in a Box

Obtain the energy spectrum of relativistic fermions of mass m placed in external constant magnetic field B (along z-axis). Suppose the system to be of a finite size l along x-direction while y, z-directions are not confined.