International Theoretical Physics Olympiad - 2018

February, 2018

You have 24 hours to solve the problems, you can use any textbook and online materials published before the beginning of the contest. You can also use Computer Algebra Systems like Mathematica or Maple. However, in case you want to present solution that relies on these computations please provide a complete description and all supplementary materials including the source code.

You can upload your solutions using your team page. Although we can accept almost all possible formats, the .pdf is the preferred one. Good luck!

1 Three-body problem

Consider particles of the same mass m, with a pairwise interaction given by a repulsive potential proportional to the inverse square of the distance between them:

$$V_{ij} = \frac{A}{(r_i - r_j)^2} \quad , \quad A > 0$$

1. For a two-body problem with initial positions and velocities given by

$$\begin{aligned} &r_1 = (0,\ 0,\ -d) \quad , \quad v_1 = (0,\ 0,\ v) \\ &r_2 = (0,\ 0,\ 0) \quad , \quad v_2 = (0,\ 0,\ 0) \\ &d > 0 \quad , \quad v > 0 \end{aligned}$$

   you are asked to find the particle trajectories.

2. Now, for a three-body problem with an additional particle

$$r_3 = (0,\ 0,\ d) \quad , \quad v_3 = (0,\ 0,\ v)$$

   find the final velocities.

2 Polarized radiation

Consider a hot rotating spherical body in vacuum. It radiates electromagnetic waves which are expected to be polarized. Find the spatial dependence of the polarization far away from the radiating body. You may find it convenient to start with a cylindrically symmetric rotating thermal radiation.

3 Dynamical mass

A free massive scalar field ϕ in $1 + 1$ dimensions is described by

$$\mathcal{L} = (\partial_t \phi)^2 - (\partial_x \phi)^2 - m^2 \phi^2$$

In many physical problems it is useful to promote coupling constants to dynamical fields. We can do the same with the mass m considering the following partition function:

$$Z = \int \mathcal{D}\phi \mathcal{D}m \ \exp\left(i \int dtdx \ (\partial_t \phi)^2 - (\partial_x \phi)^2 - m(x)^2 \phi^2\right)$$

Study the stability of the following vacuum: $\phi = 0,\ m = m_0 = const \neq 0$. Consider two cases: infinite volume and a finite interval $[0, L]$ in the x direction with Dirichlet boundary conditions for ϕ and free boundary conditions for m.

4 Isings

In this problem you are asked to calculate analytically or numerically the free energy as a function of temperature for the following three models:

1. The 2-dimensional Ising Model with an interaction

   $$\mathcal{E} = \sum_{\hat{ij}} \sigma_i \sigma_j, \quad \sigma_i = \pm 1$$

   Where ij are adjacent vertexes.

2. The 3-dimensional Ising Model with an interaction

   $$\mathcal{E} = \sum_{\hat{ij}} \sigma_i \sigma_j, \quad \sigma_i = \pm 1$$

   Where ij are adjacent vertexes.

3. The 3-dimensional tensor model with an interaction

   $$\mathcal{E} = \sum_{abc, a'b'c'} \sigma_{abc} \sigma_{ab'c'} \sigma_{a'bc'} \sigma_{a'b'c}, \quad \sigma_{abc} = \pm 1$$

   Does this model experience phase transition? If yes, what is the order of this transition?

5 Fermionic lobsters

Peter The Experimentalist caught a finite number of lobsters in Maine. He decided first to play with them before having a nice dinner. Peter put these lobsters on an one-dimensional lattice. It is forbidden for lobsters to be at the same site of the lattice at the same moment. After that Peter waits N minutes, during a minute only one lobster can make only one hop to the left or to the right. Peter wants to know the possible final states with their multiplicities if the lobsters are allowed to hop only to the right $\{n_i^R\}$ or to the left $\{n_i^L\}$. You are asked to prove or disprove the following statements:

1. $\sum n_i^L = \sum n_i^R$
2. $\sum \left(n_i^L\right)^2 = \sum \left(n_i^R\right)^2$
3. If we are given a pattern of moves L or R and also the possible final states with multiplicities $\{n_i^{\text{arb}}\}$, then $\sum \left(n_i^{\text{arb}}\right)^2$ doesn't depend on the choices L or R but on the number of hops.