International Theoretical Physics Olympiad

January 2019

You have 24 hours to solve the problems below. You can use any textbook and online materials published before the beginning of the contest, provided that you cite your references appropriately. You can also use Computer Algebra Systems like Mathematica or Maple. However, in case you want to present a solution that relies on these computations, please provide a complete description and all supplementary materials, including the source code. You can upload your solutions using your team page, or e-mail them to us at **info@thworldcup.com** if there are any technical difficulties. Although we can accept almost all possible formats, .pdf is preferred. Good luck!

Exercise (*1*): Bendy plates.

A culinarily challenged amplitudologist decided to bake some cookies. To his horror, he discovered that his baking sheet has spontaneously broken a Z_2 symmetry: a formerly flat metal sheet, it would bulge up and down, picking one of the two vacua under a slight application of pressure.

Why would a metal sheet bulge like that? Why was the symmetry broken? Devise a toy model for the baking sheet that would exhibit this symmetry-breaking property. Make whatever assumptions you find reasonable.

Exercise (*2*): Cosmological vertigo.

There exist vacuum geometries which are partially expanding (positive cosmological constant, c.c.) and partially collapsing (negative c.c.). Find such a geometry, and write it the following form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + R(r,t)^2 d\Omega^2 \tag{1}$$

Where $d\Omega^2$ are some additional degrees of freedom warped by $R(r,t)$. Explain why you think your solution satisfies the criterion requested. Then, add in a spherical shell of massive particles with infinitesimally low energy density at the interface(s) of the expanding and collapsing regions of the spacetime. What criterion must you now impose on a generic positive-c.c. bubble of your universe?

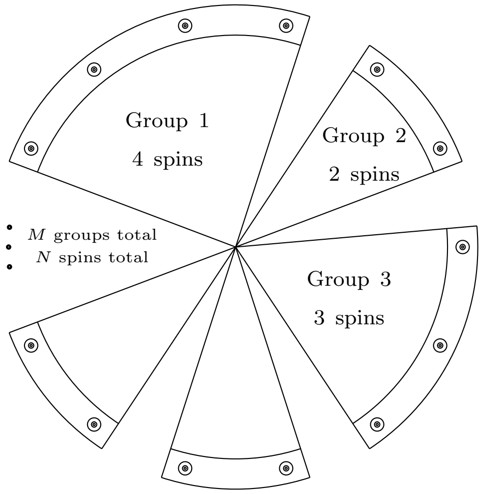

Figure 1: Schematic for Exercise 3.

Exercise (*3*): Disentanglement algorithms.

Suppose you have a chain of N spins placed adjacent to one another around a circle, as in Figure 1. Some of these spins may be entangled within M subchains of indefinite length, where $1 \leq M \leq N$. You are provided with the density matrix ρ corresponding to the system.

1. Construct an efficient algorithm which will isolate the M subchains of the system. Assume that these subchains will be visible in the form of ρ within a finite number b of bases for your Hilbert space. Include an analysis of the complexity of your algorithm. Do so assuming that you have crafted this spin chain in a laboratory so that entangled subchains will consist only of adjacent spins.

2. A competing scientist sees that you are close to a result on your spin chains breakthrough paper and, in a dastardly attempt to jeopardize your grant funding, decides to rearrange the spins in your apparatus. Thankfully, this evildoer does so without disrupting the entanglement of the spins. Modify your algorithm to relax the assumption that subchains consist of adjacent spins. You may now encounter troubling exponential runtimes, so try your best to reduce them.

Exercise (*4*): Colony collapse in a nutshell.

Bacteria, such as E. coli, can collectively look for food via a chemotaxis signaling network. When bacteria eat food, they produce attractant molecules in the environment (a liquid medium). By sensing the gradient of attractant concentration, bacteria (in a run-and-tumble motion) swim as random-walkers with bias.

A system of PDEs – the Keller-Segel equations – is often used to describe bacteria chemotaxis:

$$\partial_t b - D_b \nabla^2 b + \kappa \nabla (b \nabla c) = \alpha b \tag{2}$$

$$\partial_t c - D_c \nabla^2 c = \beta b f \tag{3}$$

$$\partial_t f - D_f \nabla^2 f = -\gamma b \tag{4}$$

Here b stands for the bacterial density, c for the attractant concentration, and f for food concentration. The parameters D_b, D_c and D_f are the diffusivity coefficients of the bacteria, the chemoattractant molecules and the food molecules. The parameter κ is the sensitivity of bacteria to the chemo concentration gradient, α is the bacterial growth rate, β is the chemo production rate, and γ is the food consumption rate.

Consider the dynamics of a bacterial colony in an environment of volume V with a hollow shell in the center (volume $V_s \ll V$). Let the hollow shell have a small hole (opening area A_s) on it to connect the inside and the outside. Assume that in the beginning the food concentration is the same everywhere, and we inoculate some bacteria into the environment. Your preliminary experiments show that the bacterial colony collapses into the hollow shell. Model this behavior theoretically and numerically. You may need to modify the Keller-Segel equations.

Exercise (*5*): Hot and heavy.

It is well-known that in a flat spacetime a finite temperature can lead to the screening of electric fields (in Debye mass phenomena, the field A_0 acquires a mass). It is also known that in Rindler spacetime there is a finite temperature due to quantum effects. Does this temperature lead to the emergence of Debye screening in Rindler space? If so, why?

Exercise (*6*): Resistance is futile.

Compute the mean effective resistance $\bar{R}_{AB}$ of a statistical mechanical ensemble of random graphs of resistors connecting two points A and B in a two-dimensional plane. Assume that each vertex has exactly 4 edges of constant resistance R extending from it. When you are done, compute the mean effective spring constant assuming the resistors are now springs.