International Theoretical Physics Olympiad

January, 2020

You have 24 hours to solve the problems below. You can use any textbook and online materials published before the beginning of the contest, provided that you cite your references appropriately. You can also use Computer Algebra Systems like Mathematica or Maple. However, in case you want to present a solution that relies on these computations, please provide a complete description and all supplementary materials, including the source code. You can upload your solutions using your team page, or e-mail them to us at **info@thworldcup.com** if there are any technical difficulties. Although we can accept almost all possible formats, .pdf is preferred. Good luck!

Exercise (*1*): Tempest in a Teapot.

In principle, boiling water can source gravitational waves. State why that is, and describe how gravitational waves may be produced in a teapot with boiling water. Estimate the peak of the power spectrum of gravitational waves produced in this process. Make and justify whatever assumptions you find reasonable, and always cite your sources.

Exercise (*2*): Wiley Invariance.

Consider a free, massless scalar field ϕ in two-dimensional Minkowski spacetime. Show that the action is invariant under the transformation rule

$$\phi \mapsto \phi + \delta\phi \equiv \phi - \varepsilon(z)\partial\phi - \bar{\varepsilon}(\bar{z})\bar{\partial}\phi \tag{1}$$

Where z and $\bar{z}$ are the so-called "lightcone coordinates" in two dimensions, ∂ and $\bar{\partial}$ are their corresponding derivatives, and $\varepsilon(z)$ and $\bar{\varepsilon}(\bar{z})$ are small but arbitrary functions of the indicated coordinates. Next, for a small parameter α consider the following Lagrangian density, as an extension of the free scalar:

$$\mathcal{L} = \partial\phi\bar{\partial}\phi + \alpha(\partial\phi\bar{\partial}\phi)^2 \tag{2}$$

Is it possible to add $\mathcal{O}(\alpha)$ contributions to $\delta\phi$ which are first-order in ϕ-derivatives and which preserve this invariance?

Exercise (*3*): Swiss Cheese Universe.

Write down a spacetime metric corresponding to a classical black hole system in an infinite number of spacial dimensions. Using your result, generate what is known as a *multi-center* black hole geometry. More specifically, describe an infinite-dimensional spacetime which has an arbitrary number of black holes interspersed at arbitrary locations.

Now relax the limit of infinite dimensions and consider this spacetime in a large enough number of dimensions d so that the geometry is nearly identical. Distribute a number density ρ of these black holes approximately uniformly across some d-sphere of finite radius R. Analytically or numerically, model the density of a diffusive gas of massive particles as it moves through this spacetime, provided the initial density of the gas is Gaussian-distributed with width $\ell \ll R$ and a small total mass M.

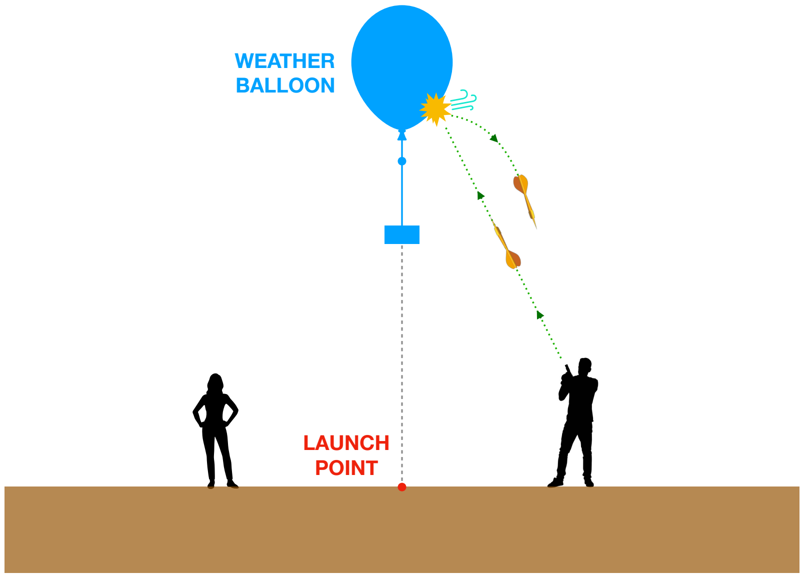
Exercise (*4*): Bubble, Bubble, Euler Trouble.

When spherical air bubbles are released underwater, they drift toward the surface with a velocity which depends on their depth and their volume, among other physical criteria.

1. Model such a spherical bubble and calculate its upward drift velocity. Then, modify your model and compute the same quantity for a bubble of genus 1 – a “bubble ring.” Assume the plane of the ring is approximately parallel to the water’s surface.

2. Is it possible to make any comments (qualitative or quantitative) on the drift velocity of higher-genus bubbles?

3. Dolphins are known to play with each other by forming and manipulating stable bubble rings. How can the statement of this problem be modified so that the ring sits in a stable configuration underwater?



Exercise (*5*): Balloon Wars.

Two actively hostile high-altitude balloon experiments aim to launch a weather balloon. One of the experiments succeeds, and, in unusually opportune windowless conditions, the balloon begins to rise vertically upward. The payload attached to the balloon ensures it does not rotate. When the other group's PI sees the balloon, in a rage of uncontrollable jealousy, he shoots an air dart at the balloon, making a small spherical opening on its surface.

1. Predict the balloon's trajectory. Assume that initially, the thrust is much larger than drag, and that the balloon is low enough so that the surrounding air density does not vary appreciably as the balloon descends.

2. The other experiment still wants to save their balloon and their precious data. Can they also shoot an air dart gun at their balloon, once or several times, so that the balloon descends down as close as possible to its point of launch? How does this answer depend on where the initial dart strikes the balloon's surface, and on how soon after the first strike they respond?

Exercise (*6*): Lazy physicist.

In a simple approximation, a physicist's office could be described as a room with a desk in it. Because the physicist is really busy with teaching and submitting papers, they do not regularly remove the stacks of papers accumulating on the desk. This, unfortunately, implies that they cannot remove the dust on the desk. On the other hand, they hoover the floor of the office once a week. Making reasonable assumptions about the dust production, dust properties, airflow, geometry, etc, estimate the average amount of dust to be found on the desk in the steady state.

Optional suggestions:

- You can model the rate of dust deposition in the office as constant in time and space.
- The airflow may be approximated as highly chaotic and its moments as constant in time, thus the macroscopic motion of the dust particles induced by the airflow is equivalent to an effective temperature (you may assume the Brownian motion due to the actual temperature to be negligible).
- It is reasonable to assume a contact interaction between the dust particles and the floor as well as the desk surface (i.e. the chemical potentials are shifted by a constant term).
- You may neglect the effect of the papers on the desk.
- If you want to make the model even more realistic, you can add another interaction term proportional to the dust density on floor and desk surface, this models the short-range interaction between different dust particles (which you may assume to be negligible in the phase in which dust particles move freely in the air, due to their low density).
- You may assume the desk surface area to be small in comparison to the cross-section of the office whenever this approximation yields simplifications.