Storm in a bar

The core idea behind the present problem is essentially the variation of the Einstein-de Haas effect. According to this effect, changes in magnetization lead to changes in mechanical angular momentum. There are no other possible sources of angular momentum (contribution from electromagnetic radiation is prohibited by symmetry). Hence the rest of the problem lies in figuring out the collision-induced change in the magnetization of the rod.

The physics of real ferromagnets is challenging due to the presence of domain walls and the hysteretic behavior (that leads to different possible states of the rod at a given temperature). None of the teams managed to fully describe the domain-wall-related physics, so we decide to focus on the simplest approach to the problem, focusing on the core physics related to Einstein-de Haas effect. Another interesting aspect of the problem is the collision and generated acoustic waves. This part of the problem is relevant for equilibration dynamics, relaxation to the quasiequilibrium state right after the collision (both with or without consideration of the domain walls).

EINSTEIN-DE HAAS EFFECT

As we have mentioned in the introduction, the core idea behind the problem is essentially the variation of Einstein-de Haas effect

$$\Delta \boldsymbol{L} = -\gamma \Delta \boldsymbol{m}, \tag{1}$$

where γ is a gyromagnetic ratio and $\boldsymbol{m} = V\boldsymbol{M}$ is the magnetic dipole moment of the rod, while $\boldsymbol{M}$ and V are the magnetization and the volume of the rod. Collision leads to changes in the magnetization of the rod which in turn leads to mechanical angular momentum.

HEATING AFTER THE COLLISION

After the collision a fraction of kinetic energy is converted into internal energy of the rod. Inelastic collisions are often described by the coefficient of restitution,

$$c_{\mathrm{r}} = \frac{\text{final kinetic energy}}{\text{initial kinetic energy}}. \tag{2}$$

In other words, we can find the change in temperature from

$$C\Delta T = k \cdot \frac{mv^2}{2} \implies \Delta T = \frac{kmv^2}{2C}, \tag{3}$$

where $k = 1 - c_{\mathrm{r}}$ and C is the heat capacity.

WEAK COLLISION

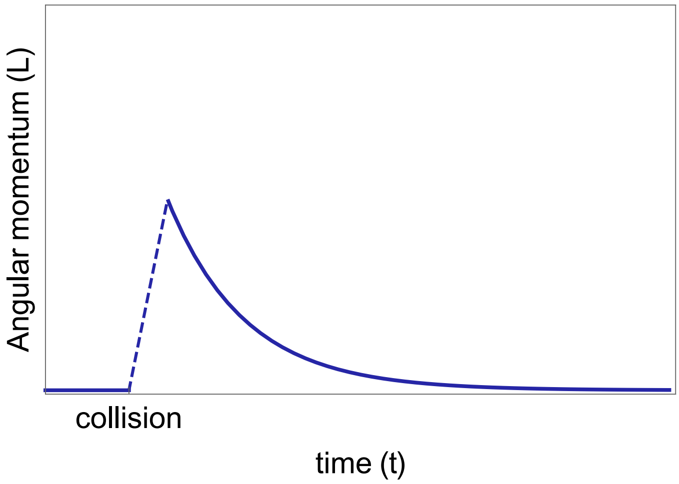
If the collision is sufficiently weak, magnetization changes only by a small amount, according to

$$\Delta M = \frac{\partial M}{\partial T} \Delta T, \tag{4}$$

and then slowly relaxes to equilibrium with the medium. Hence right after the collision the angular momentum is

$$L = \gamma V \left(\frac{\partial M}{\partial T} \right) \cdot \frac{kmv^2}{2C}. \tag{5}$$

After a long time the magnetization M relaxes back to initial equilibrium pre-collision value while angular momentum goes to zero.



STRONG COLLISION

If the collision is strong (i.e. $\Delta T > T_c - T$), then magnetization may disappear completely yielding angular momentum

$$L = \gamma V M_0. \tag{6}$$

After the collision the rod slowly cools down back to the transition temperature. When the spontaneous symmetry breaking occurs, to the leading order both possible directions are equally likely so that the final magnetization is $\pm M_0$ with final angular momentum

$$L = \begin{cases} 0 & 50\% \\ 2\gamma V M_0 & 50\% \end{cases} \tag{7}$$

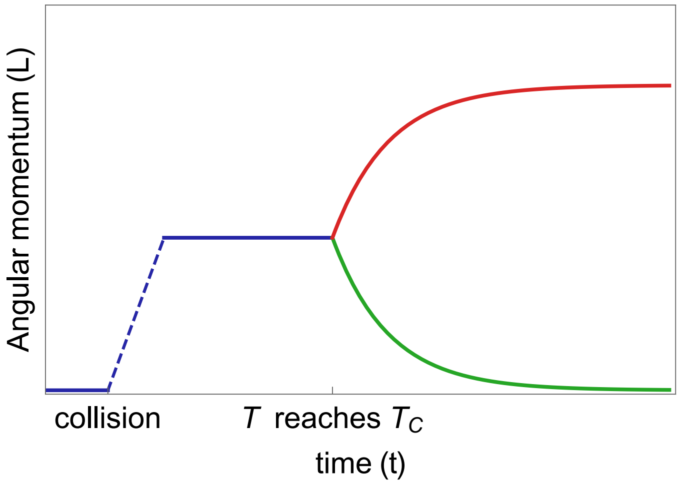


FIG. 1: When the rod cools down to the T_c after the collision it has equal chances to redevelop initial magnetization M_0 leaving no angular momentum (green line) or develop magnetization in an opposite direction $-M_0$, leading to angular momentum $L_0 = 2\gamma M_0$ (red line).

Of course in real medium there are always drag forces (unless your experimental setup is immersed in liquid helium), so in the long run even leftover magnetization $2\gamma V M_0$ disappears.