3 Dynamical mass

(idea by A. Milekhin)

A free massive scalar field ϕ in $1+1$ dimensions is described by

$$\mathcal{L} = (\partial_t \phi)^2 - (\partial_x \phi)^2 - m^2 \phi^2$$

In many physical problems it is useful to promote coupling constants to dynamical fields. We can do the same with the mass m considering the following partition function:

$$Z = \int \mathcal{D}\phi \mathcal{D}m \ \exp\left(i \int dtdx \ (\partial_t \phi)^2 - (\partial_x \phi)^2 - m(x)^2 \phi^2 \right)$$

Study the stability of the following vacuum: $\phi = 0, \ m = m_0 = const \neq 0$. Consider two cases: infinite volume and a finite interval $[0, L]$ in the x direction with Dirichlet boundary conditions for ϕ and free boundary conditions for m.

Solution:

To check if the configuration $\phi = 0, m = m_0 = const$ is indeed vacuum, first we need to assume that at infinity $\phi \to \phi_{cl}$ and m is constant. Then we will investigate the form of the effective potential for ϕ_{cl} and m. Integrating out ϕ by computing $\det(\partial^2 + m^2)$ produces the celebrated Coleman–Weinberg effective potential:

$$V_{eff}(m, \phi_{cl}) = m^2 \log \frac{m^2}{M^2} - m^2 + m^2 \phi_{cl}^2 \tag{1}$$

where M is the renormalization scale. The potential has minimum at $m = 0$ and $\phi_{cl} = -\infty$. However, at $m = 0$ the theory has symmetry $\phi \to \phi + const$, therefore ϕ_{cl} can be renormalized to any value. For example to $\phi_{cl} = 0$.

On the infinite line because of the translation invariance it is obvious that configuration with constant m and ϕ is stable under non-uniform perturbations.

However, on a finite interval with Dirichlet boundary conditions the translation invariance is broken. It turned out that it is quite hard to find the exact $m(x)$ profile, however it is easy to show that $m(x) = const$ *is not even a saddle-point* of the effective potential. We refer to the original paper [1](see also [2]) for the derivation.

References

[1] S. Bolognesi, K. Konishi and K. Ohashi, JHEP **1610**, 073 (2016) doi:10.1007/JHEP10(2016)073 [arXiv:1604.05630 [hep-th]].

[2] A. Milekhin, Phys. Rev. D **95**, no. 8, 085021 (2017) doi:10.1103/PhysRevD.95.085021 [arXiv:1612.02075 [hep-th]].