ITPO 2019, Problem 2: Cosmological Vertigo

March 2, 2019

Problem Statement:

There exist vacuum geometries which are partially expanding (positive cosmological constant, c.c.) and partially collapsing (negative c.c.). Find such a geometry, and write it the following form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + R(r,t)^2 d\Omega^2 \tag{1}$$

Where $d\Omega^2$ are some additional degrees of freedom warped by $R(r,t)$. Explain why you think your solution satisfies the criterion requested. Then, add in a spherical shell of massive particles with infinitesimally low energy density at the interface(s) of the expanding and collapsing regions of the spacetime. What criterion must you now impose on a generic positive-c.c. bubble of your universe?

Rubric:

	Criterion	Point value	Total
Part A: Find f(r)	Identified dS vs. AdS	2	10
	Lambda switches sign	4	
	Smooth interpolation	2	
	Absolute value used	2	
Part B: Justify	Truly vacuum / dilaton vacuum	4	5
	(A)dS2 x S2	1	
Part C: Spherical shell	Reasonable discussion	3	5
	dS behind SAdS horizon	2	

Solution:

There are many such geometries which exhibit this behavior. The key to the first part of the problem is to note that flipping the sign of the cosmological constant swaps "Anti de Sitter" (AdS) geometries for "de Sitter" (dS) geometries and vice versa. In 1+1 dimensions, those metrics look like:

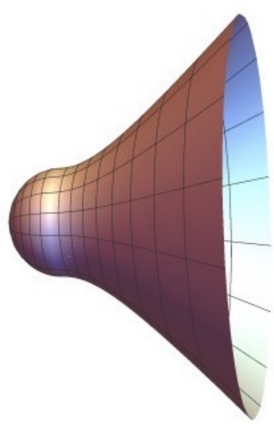
$$ds^2_{AdS} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} \tag{2}$$

$$ds^2_{dS} = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} \tag{3}$$

If we want to smoothly join these (which is a necessary condition for a vacuum theory, as kinks imply excitations), we can simply set $f(r) = 1 + \frac{r^3}{|r|}$, and consider $r > -1$. For negative r the geometry is the so-called "static patch" of de Sitter, which transitions smoothly to an Anti de Sitter geometry at $r = 0$. This is what we call a "Centaur geometry":

$$ds^2 = -\left(1 + \frac{r^3}{|r|}\right)dt^2 + \frac{dr^2}{1 + \frac{r^3}{|r|}} \tag{4}$$

The geometry proposed above has the following Euclidean slice (image courtesy of [1]):



We call it a "centaur" because in the Euclidean picture, it involves sewing half of a dS geometry with half of an AdS one. As described in the original paper on the subject [1], it is a particular vacuum solution of the following 2D dilaton gravity theory:

$$S = \frac{\phi_0}{16\pi G}\int d^2x\sqrt{-g}R - \frac{1}{16\pi G}\int d^2x\sqrt{-g}\Big(\phi R + \ell^2 V(\phi)\Big) \tag{5}$$

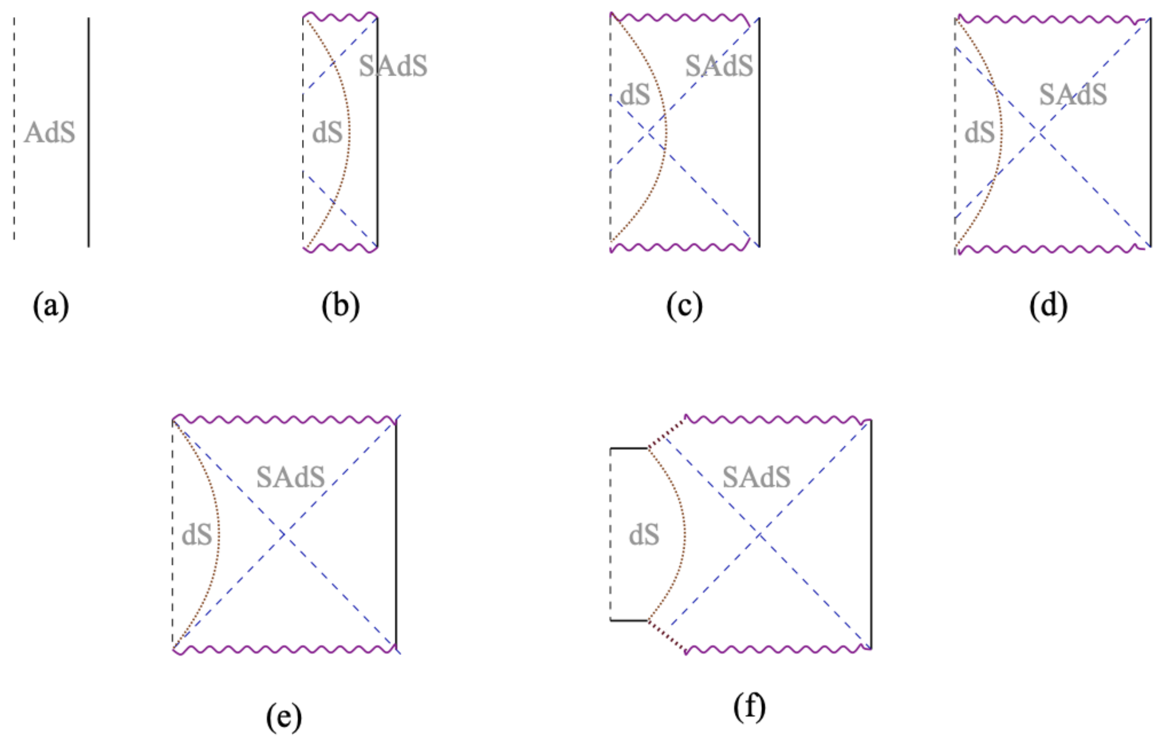
Where ϕ_0 is the constant background of some dilaton field Φ, with ϕ the fluctuations about the background. Here we can take $V(\phi) = 2(\sqrt{\phi^2+\epsilon^2} - \epsilon)$ with ϵ a parameter which tunes the infrared scale $\ell\epsilon$ and which demarcates the boundary between AdS and dS behaviors.

Now imagine instead that we merge the two geometries across a thin spherical domain-wall of massive particles. This is very analogous to the physical picture of thin-walled Coleman-de Luccia (CdL) false vacuum bubbles. It is possible to show that there is now a "jump condition" which must be satisfied by the geometries on either end of the domain-wall. This condition will be parameterized by the extrinsic curvature κ and radius $R(t)$ of the spherical shell:

$$ds^2_{wall} = -dt^2 + R(t)^2 d\Omega^2 \tag{6}$$

$$\kappa R = \sqrt{\dot{R}^2 + f_{dS}(R)} - \sqrt{\dot{R}^2 + f_{AdS}(R)} \tag{7}$$

As shown in [2], in this physical picture a dS bubble which forms in a time-symmetric universe must now be placed behind the event horizon of an AdS-Schwarzschild black hole![1] We will walk through the intuition for a particular time-symmetric case. Consider the following sequence of Penrose diagrams, courtesy of [2]:



(a) This is the Penrose diagram for our initial AdS geometry. The solid line at the right is a spacial (timelike) boundary $r = \infty$. The dashed line at the left is an $r = 0$ dS boundary.

(b) If we add in the spherical shell of particles (dotted line) so that it initially expands and

[1] For time-asymmetric configurations the answer changes somewhat, but generally the dS bubble is causally disconnected from one of the Rindler wedges (the right triangle with hypotenuse $r = \infty$) of the AdS geometry. This is more difficult to show, and solutions which made comments to this end earned extra points.

collapses (again, in a time-symmetric fashion), its infinitesimal energy density requires the bubble to form from and collapse into a singularity (squiggly lines). Our AdS space becomes AdS-Schwarzschild (SAdS).

(c) The prior scenario is not generically possible for all SAdS black holes because the extrinsic curvature of the spherical shell (which should be constant in this construction) can change sign in the region where it moves through the past horizon. We therefore begin to expand the SAdS spacetime until the construction is logically consistent.

(d) As in the prior part, we must expand the SAdS geometry until the dS shell moves past the "bifurcation point" at the center, where the past and future horizons (diagonal, dashed lines) meet.

(e) To prevent the dS bubble from crossing additional horizons, we must shrink it down to lie within the left Rindler wedge (the right triangle with $r = 0$ as its hypotenuse).

(f) Now, if we wish, we can add in dS asymptotic regions (spacelike boundaries, horizontal solid black lines) so long as we join the boundaries with some appropriate physical merging condition (depicted as diagonal dotted lines). We effectively have a dS universe living within an SAdS black hole.

References

[1] D. Anninos and D. M. Hofman, "Infrared Realization of dS_2 in AdS_2," *Class. Quant. Grav.* **35** no. 8, (2018) 085003, `arXiv:1703.04622 [hep-th]`.

[2] B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers, M. Rangamani, and S. Shenker, "Inflation in AdS/CFT," *JHEP* **03** (2006) 007, `arXiv:hep-th/0510046 [hep-th]`.