ITPO 2020 Solutions

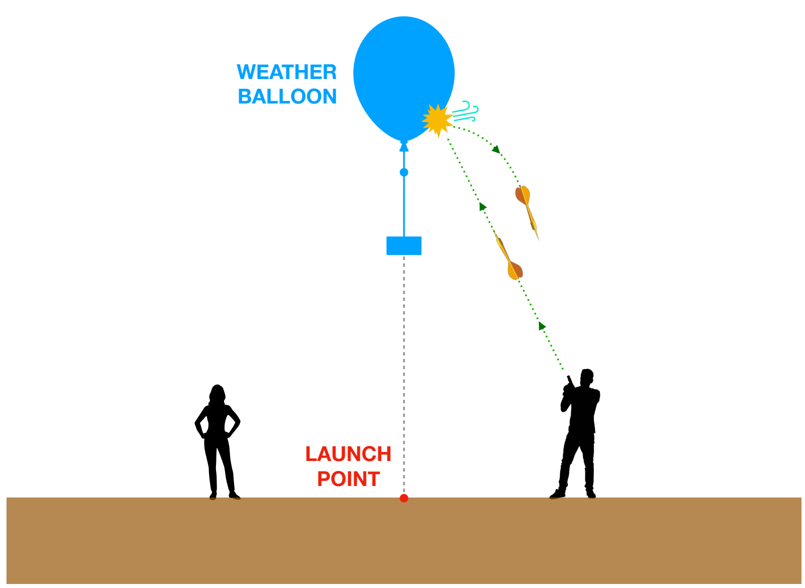
Problem 5

Exercise (*5*): Balloon Wars.

Two actively hostile high-altitude balloon experiments aim to launch a weather balloon. One of the experiments succeeds, and, in unusually opportune windowless conditions, the balloon begins to rise vertically upward. The payload attached to the balloon ensures it does not rotate. When the other group's PI sees the balloon, in a rage of uncontrollable jealousy, he shoots an air dart at the balloon, making a small spherical opening on its surface.

1. Predict the balloon's trajectory. Assume that initially, the thrust is much larger than drag, and that the balloon is low enough so that the surrounding air density does not vary appreciably as the balloon descends.

2. The other experiment still wants to save their balloon and their precious data. Can they also shoot an air dart gun at their balloon, once or several times, so that the balloon descends down as close as possible to its point of launch? How does this answer depend on where the initial dart strikes the balloon's surface, and on how soon after the first strike they respond?



Rubric:

	Criterion	Point value	Total
	Reasonable equation of state for gas inside the balloon	3	
Part A:	Reasonable equation of state for the balloon	3	
	Reasonable attempt to model the balloons thrust	8	18
Part B:	Numerical modelling	4	
	Reasonably extensive analysis within a given model	2	2

Comments:

The major point of this problem was to illustrate the degree of complexity separating first-principles physical reasoning and real-world applications. A successful solution to this problem needed to combine first principles, phenomenological approaches, and numerical modeling.

As far as phenomenological assumptions go, we were quite lenient with allowing a wide range of models, with a reasonable restriction: models that predict linear or parabolic motion are obviously not sufficiently sophisticated. Even with the assumptions explicitly allowed by the problem, the physics of the balloon's thrust will certainly generate complicated trajectories. Physical intuition is important. From the standpoint of model design, sometimes it helps to work back from your model's predictions to judge whether your assumptions are clearly too stringent. Because of this, a number of assumptions were deemed excessively restrictive, such as

- making the gas flow incompressible (or applying Bernoulli's equation);
- giving exhaust a linear velocity; or
- assuming constant pressure inside the balloon.

A number of equations we explicitly looked for in the solutions were:

- The equation of state for the gas inside the balloon;
- The equation of state for the balloon, such as the connection between its pressure and volume; and, most importantly,
- A model for the balloon's thrust.

Absence of or a deficiency in any of the three of these led to a deduction of points according to the rubric.

For the purposes of a simple model, the equation of state for the gas was allowed to be that of an ideal gas at constant temperature.

The equation of state for the balloon could have a number of phenomenological realizations.

As a suggestion, consider the physical intuition that inflating a balloon is hard initially, but then gets progressively easier until the very end, when it gets hard again just before the balloon pops. This intuition suggests that within the regime of interest, the pressure in the balloon decreases as the balloon deflates.

Lastly, the modeling of the balloon's thrust was perhaps the most challenging element of the problem. As mentioned above, we were quite lenient with a wide range of assumptions, apart from those that lead to obviously oversimplified predictions.

Another important component of this problem was a numerical experiment. Numerical component is essential in modeling, in terms of both understanding your model and presenting it to others. For this reason, an analytic derivation without a numerical component lost a number of points according to the rubric.

While usually multi-part problems at ITPO get harder in each subsequent part, this one was an exception. However, a simple statement of a number of salient points (such as the importance of reaction time) was insufficient. A satisfactory response to this part of the problem had to include a numerical investigation within the designed model. A good model would have lead to at least a few interesting regimes, such as the critical reaction time past which no return was possible.